Regional Frequency Analysis of Extreme Climate Events.

Theoretical part of REFRAN-CV

Course outline

- Introduction
- L-moment statistics
- Identification of Homogeneous Regions
- L-moment ratio diagrams
- Example of the Regional Frequency Analysis with the NOAA Dataset across Africa
- Bibliography

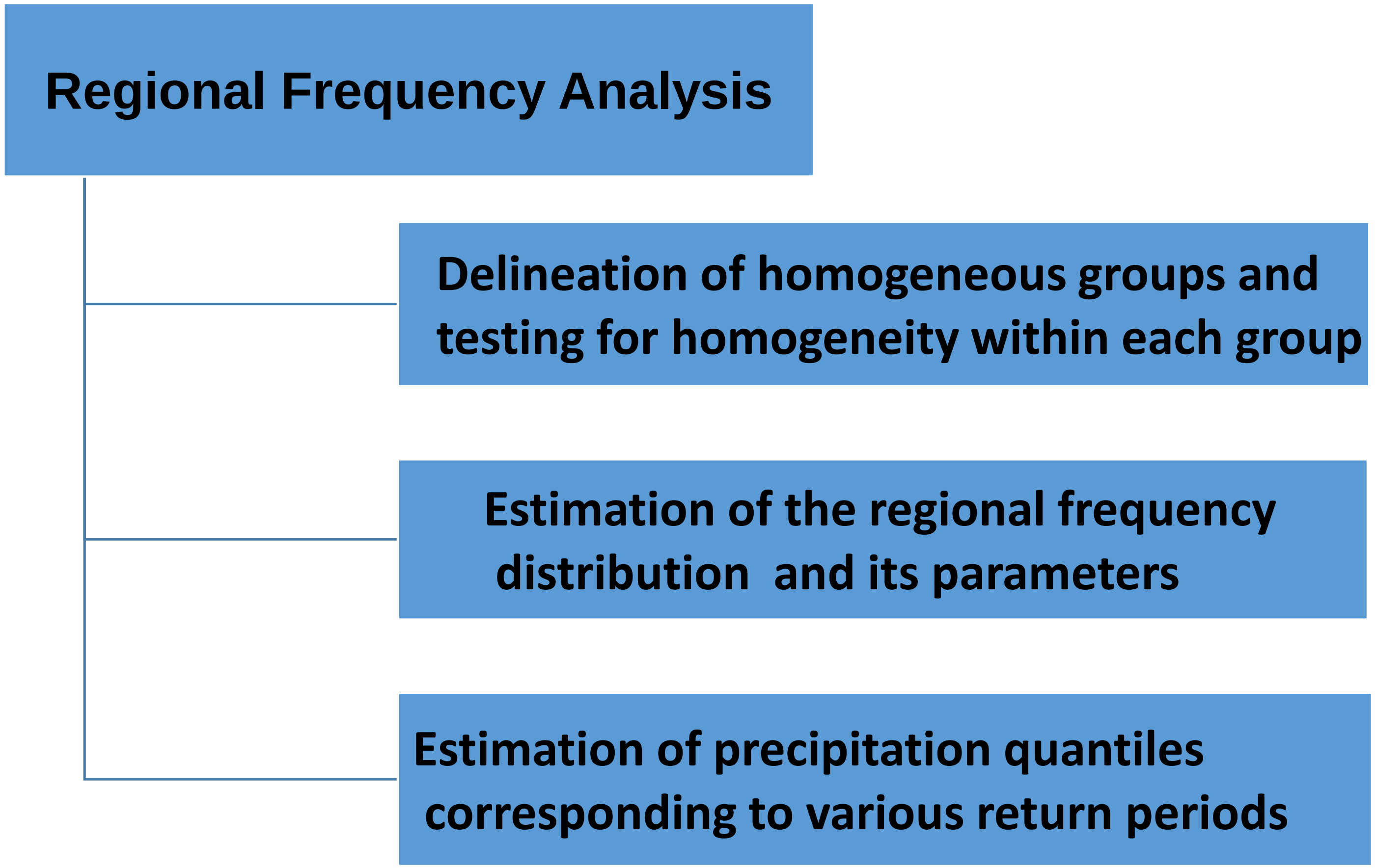
Introduction

- The estimation of extreme climate events (precipitation/temperature) can be approached with the **regional frequency analysis** [Hosking, 1990].

- Regional frequency analysis **includes information from nearly stations exhibiting similar statistical behavior as at the site under consideration** in order to obtain more reliable estimates.

Introduction

- Regional frequency analysis is applied when no local data are available at a site of interest or the data are insufficient for a reliable estimation of the required return period.



L-moment statistics

Hosking (1990) has defined L-moments to summaries theoretical distribution and observed samples. **L-moments are the pillars of the regional frequency analysis**. Let $X_{(i|n)}$ be i^{th} largest obs. in sample of size n.

- L-Moment: $$\lambda_r = r^{-1}\sum_{k=0}^{r-1}(-1)^k \binom{r-1}{k} E\left(X_{r-k:r}\right)$$

  A <u>linear</u> combination of order statistics

- Specifically, for the first 4 L-moments:

$$\lambda_1 = E\left(X_{1:1}\right)$$

$$\lambda_2 = \frac{1}{2}E\left(X_{2:2} - X_{1:2}\right)$$

$$\lambda_3 = \frac{1}{3}E\left(X_{3:3} - 2X_{2:3} + X_{1:3}\right) = \frac{1}{3}\left\{ E\left(X_{3:3} - X_{2:3}\right) - E\left(X_{2:3} + X_{1:3}\right) \right\}$$

$$\lambda_4 = \frac{1}{4}E\left(X_{4:4} - 3X_{3:4} + 3X_{2:4} - X_{1:4}\right) = \frac{1}{4}\left\{\left\{E\left[\left(X_{4:4} - X_{3:4}\right) + \left(X_{2:4} - X_{1:4}\right)\right]\right\} - 2E\left(X_{3:4} - X_{2:4}\right)\right\}$$

L-moment statistics

Finally, the **L-moment ratios** are calculated as:

L-moment mean (L-mean):

L-mean $= \tau_1 = \lambda_1$

L-moment Coefficient of variation (L-CV):

L-CV $= \tau_2 = \lambda_2/\lambda_1$

L-moment coef. of skew (L-Skewness)

L-Skewness $= \tau_3 = \lambda_3/\lambda_2$

L-moment coef. of kurtosis (L-Kurtosis)

L-Kurtosis $= \tau_3 = \lambda_4/\lambda_2$

L-moment statistics

To wrap up…

- The **L-mean** is identical to the conventional statistical mean
- The **L-cv** measures a variable's dispersion, i.e. the expected difference between two random samples
- The **L-skewness** quantifies the asymmetry of the samples distribution
- The **L-kurtosis** measures whether the samples are peaked or flat relative to a normal distribution

L-moment statistics

Advantages of L-moment approach

- Less susceptible to the presence of outliers (Because L-moments avoid squaring and cubing the data)
- Perform better with small sample sizes

Identification of Homogeneous Regions

- A **homogeneous region** is considered as an **area** within which rescaled variables in different sites **have approximately the same probability distributions**.
- All sites can be described by one common probability distribution after the site data are rescaled by their at-site mean.

Identification of Homogeneous Regions

- Homogeneous regions (grouping of sites/gages) can be determined based on the similarity of the physical and/or meteorological characteristics of the sites. This can be done by performing **cluster analysis**.
- Hosking and Wallis (1997) proposed a **statistical test for testing the heterogeneity** of the proposed homogeneous regions.
- **L-moment** statistics can then used to estimate the variability and skewness of the regional data and to test for heterogeneity as a basis for accepting or rejecting the proposed region formulation.

Identification of Homogeneous Regions

How to do it?

1. Calculate the weighted standard deviation of the at-site sample *L*-CVs,

$$V = \left\{ \sum_{i=1}^{N} n_i (\tau^{(i)} - \tau^{R})^2 \Big/ \sum_{i=1}^{N} n_i \right\}^{1/2}$$

2. Fit a four-parameter **kappa distribution** to the regional average *L*-moment ratios

$$\tau^{R}, \tau_3^{R}, \text{and } \tau_4^{R}.$$

Identification of Homogeneous Regions

How to do it?

3. Simulate a large number N_{sim} of realizations of a region with N sites, each having this kappa distribution as its frequency distribution.

4. For each simulated region, calculate V.

5. From the simulations determine mean and standard deviation of the N_{sim} values of V. Call these μ_V and σ_V

Identification of Homogeneous Regions

H : is the discrepancy between L-Moments of observed samples and L-Moments of simulated samples assessed in a series of N_{sim} Monte Carlo simulation :

$$H = \frac{V - \mu_v}{\sigma_v}$$

H≥2 : Region is heterogeneous.

1 ≤H<2 : Region is possibly heterogeneous.

H<1: Region is acceptably homogeneous.

L-moment ratio diagrams

- Once homogeneous regions are defined, a single probability distribution is applied to all sites within a homogeneous region. Thus, it is necessary to choose a best-fit distribution from a set of candidate distributions.

- The goodness-of-fit can be judged by how well the *L*-skewness and *L*-kurtosis of the fitted distribution match the regional average *L*-skewness and *L*-kurtosis of the observed data.

L-moment ratio diagrams

TABLE 18.1.2 Values of L Moments and Relationships for the Inverse of the cdf for Several Distributions

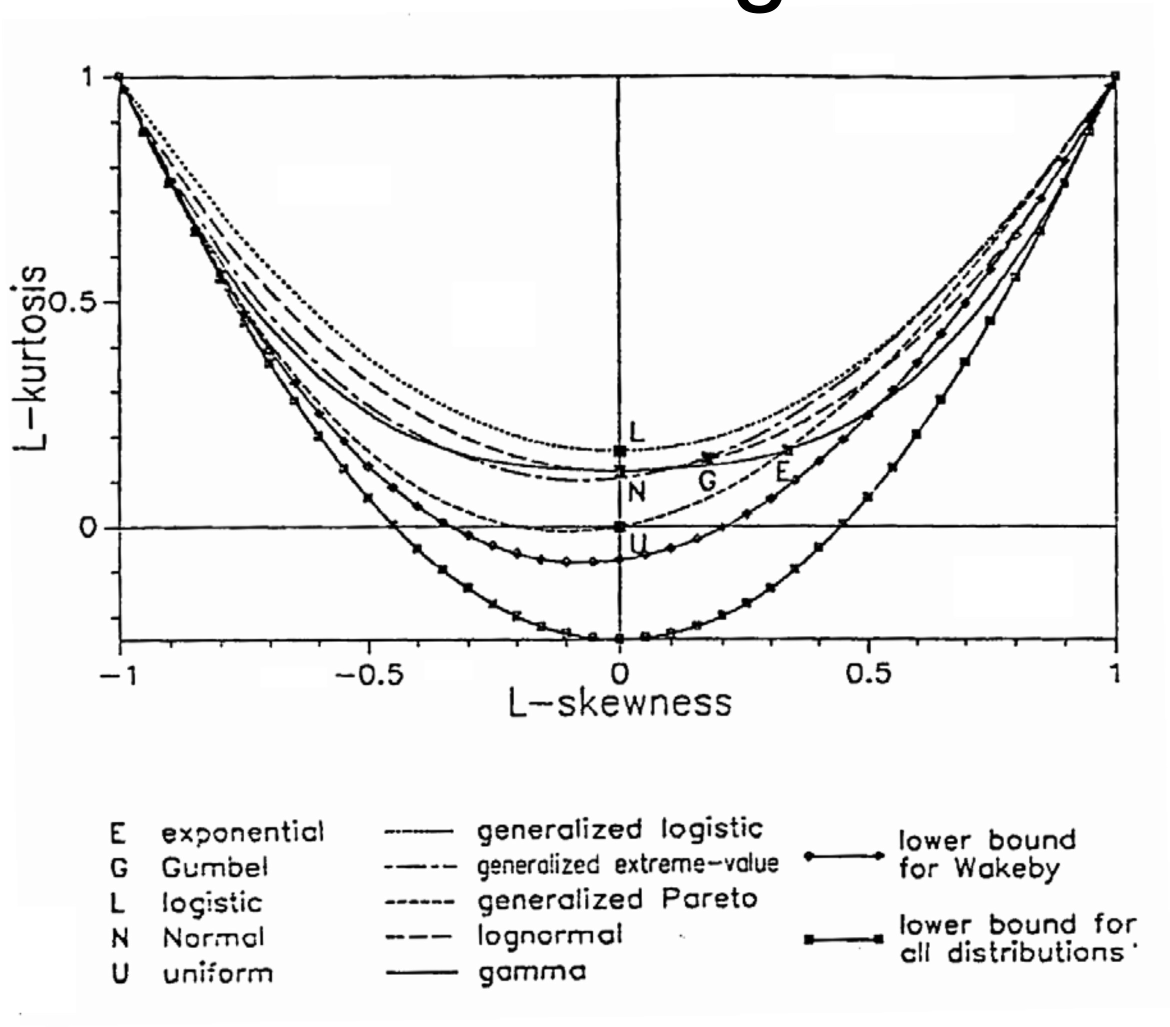
Distribution and inverse cdf	L moments
Uniform:	$\lambda_1 = \frac{\beta + \alpha}{2} \qquad \lambda_2 = \frac{\beta - \alpha}{6}$
$x = \alpha + (\beta - \alpha)F$	$\tau_3 = \tau_4 = 0$
Exponential:*	$\lambda_1 = \xi + \frac{1}{\beta} \qquad \lambda_2 = \frac{1}{2\beta}$
$x = \xi - \frac{\ln[1 - F]}{\beta}$	$\tau_3 = \frac{1}{3} \qquad \tau_4 = \frac{1}{6}$
Normal†	$\lambda_1 = \mu \qquad \lambda_2 = \frac{\sigma}{\sqrt{\pi}}$
$x = \mu + \sigma\Phi^{-1}[F]$	$\tau_3 = 0 \qquad \tau_4 = 0.1226$
Gumbel: $x = \xi - \alpha \ln[-\ln F]$	$\lambda_1 = \xi + 0.5772\,\alpha \qquad \lambda_2 = \alpha \ln 2$ $\tau_3 = 0.1699 \qquad \tau_4 = 0.1504$
GEV:	$\lambda_1 = \xi + \frac{\alpha}{\kappa}\{1 - \Gamma[1 + \kappa]\} \qquad \lambda_2 = \frac{\alpha}{\kappa}(1 - 2^{-\kappa})\,\Gamma(1 + \kappa)$
$x = \xi + \frac{\alpha}{\kappa}\{1 - [-\ln F]^{\kappa}\}$	$\tau_3 = \left\{\frac{2(1 - 3^{-\kappa})}{(1 - 2^{-\kappa})} - 3\right\}$ $\tau_4 = \frac{1 - 5(4^{-\kappa}) + 10(3^{-\kappa}) - 6(2^{-\kappa})}{1 - 2^{-\kappa}}$
Generalized Pareto:	$\lambda_1 = \xi + \frac{\alpha}{1 + \kappa} \qquad \lambda_2 = \frac{\alpha}{(1 + \kappa)(2 + \kappa)}$
$x = \xi + \frac{\alpha}{\kappa}\{1 - [1 - F]^{\kappa}\}$	$\tau_3 = \frac{1 - \kappa}{3 + \kappa} \qquad \tau_4 = \frac{(1 - \kappa)(2 - \kappa)}{(3 + \kappa)(4 + \kappa)}$
Lognormal	See Eqs. (18.2.12), (18.2.13)
Gamma	See Eqs. (18.2.30), (18.2.31)

* Alternative parameterization consistent with that for Pareto and GEV distributions is: $x = \xi - \alpha \ln[1 - F]$ yielding $\lambda_1 = \xi + \alpha$; $\lambda_2 = \alpha/2$.
† Φ^{-1} denotes the inverse of the standard normal distribution (see Sec. 18.2.1).
Note: F denotes cdf $F_X(x)$.
Source: Adapted from Ref. 72, with corrections.

L-moments & Distribution Parameter Relations

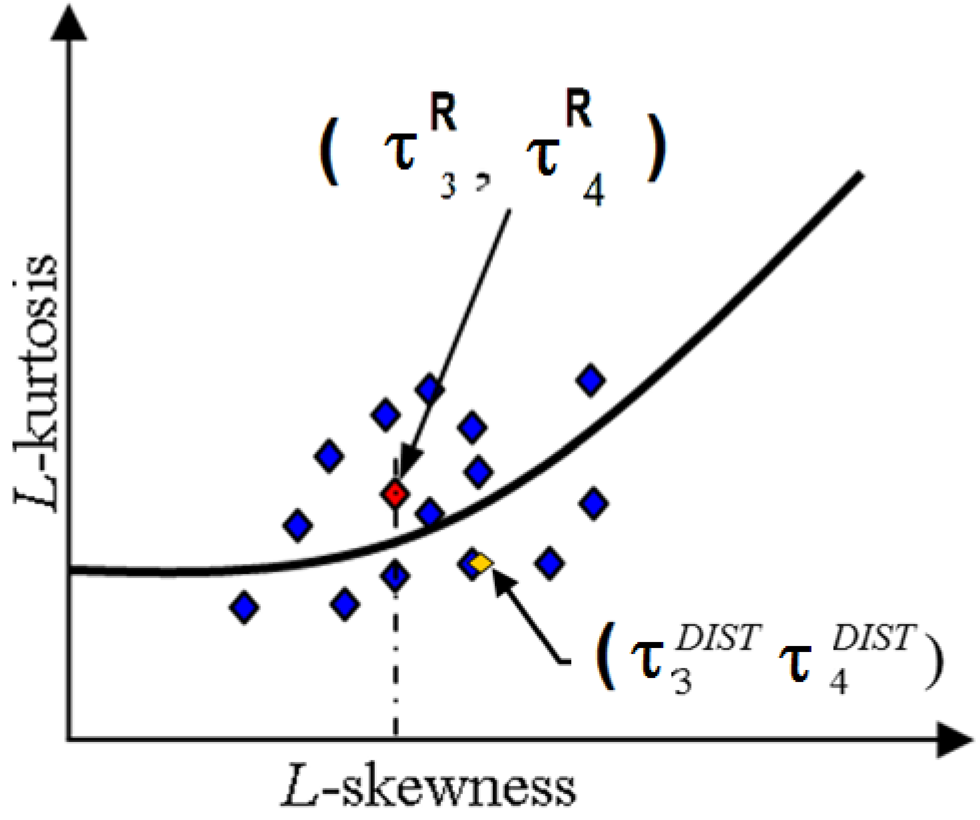
From "Frequency Analysis of Extreme Events," Chapter 8 in Handbook of Hydrology, By Stedinger, Vogel, and Foufoula-Georgiou, McGraw-Hill Book Company, New York, 1993

L-moment ratio diagrams



L-moment ratio diagrams

- The GEV distribution fitted by the *method of L-moments* has *L*-skewness equal to the regional average *L*-skewness.



- We thus judge the quality of fit by the difference between the *L*-kurtosis of the fitted GEV distribution and the regional average *L*-kurtosis.

Goodness-of-fit test

- Given a set of candidate three-parameter distributions (Pearson type III, GEV, lognormal, generalized Pareto, etc.). We need to fit each distribution to the regional average *L*-moment ratios .

$$Z^{DIST} = (\tau_4^{DIST} - \tau_4 + \beta_4)/\sigma_4$$

"Dist" refers to the candidate distribution,

τ4 DIST is the average L-Kurtosis value computed from simulation for a fitted distribution.

τ4 is the average L-Kurtosis value computed from the data of a given region,

β4 is the bias of the regional average sample L-Kurtosis,

σ_V is standard deviation.

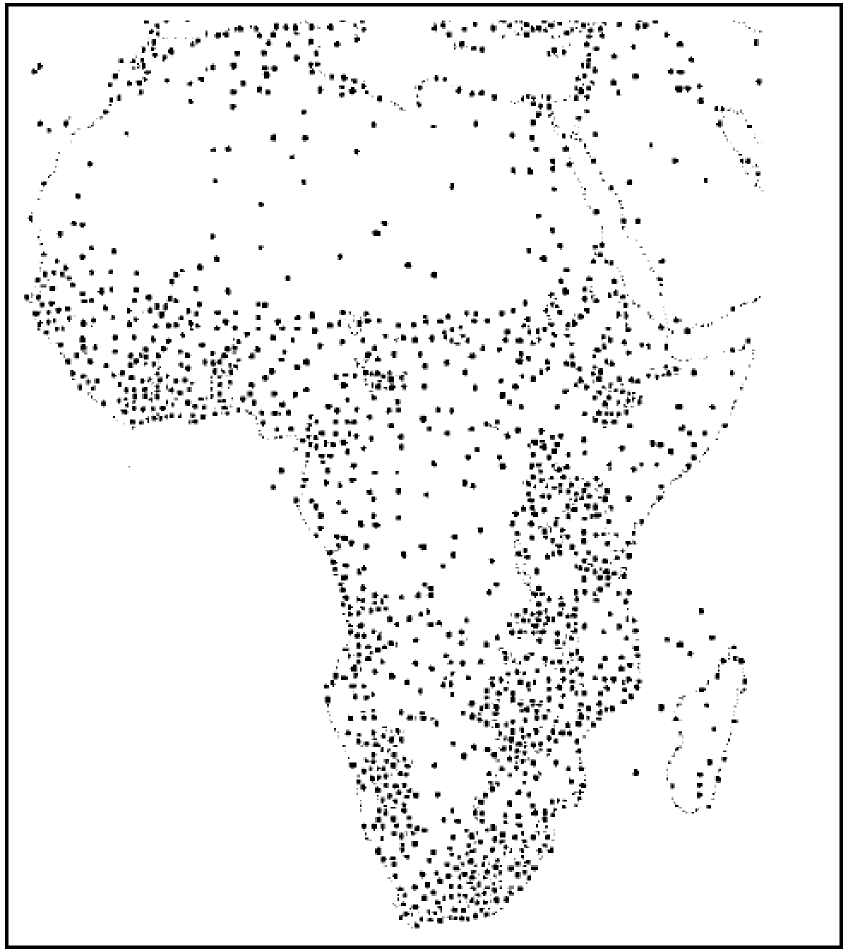
A given distribution is declared a good fit if **|ZDist|≤1.64**

Example of the regional frequency analysis with the NOAA Dataset

1. L-moments calculation at each pixel
2. Identification parameters of the precipitation probability distribution function (Annual precipitation)
3. Evaluation of the most suitable distribution function for each pixel (Annual precipitation)
4. Identification of best distribution: L-skewness and L-kurtosis were used to perform a goodness-of-fit assessment
5. Generation of precipitation maps associated with different return periods

Data coverage

NOAA's PREC/L



(Adapted from Chen et al., 2002)

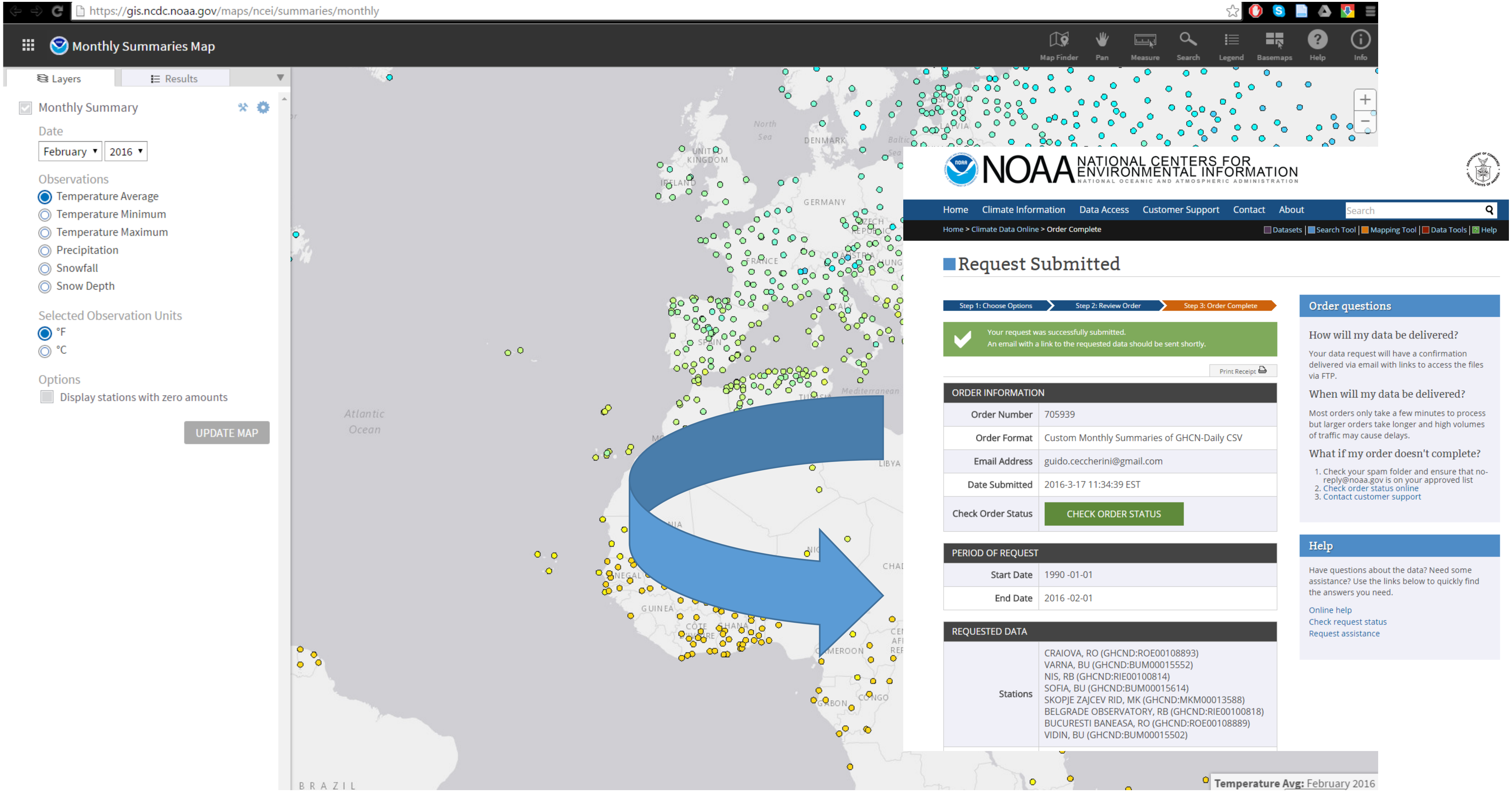
- **NOAA's Precipitation over land:**
- **Monthly records from 1951-1991 (Gauge stations with 10-yr-or-longer recording periods)**
- **Spatial resolution: 0.5 degrees**
- **Reconstruction obtained using thousands stations from the Global Historical Climatology Network (GHCN) and the Climate Anomaly Monitoring System (CAMS)**

https://gis.ncdc.noaa.gov/maps/ncei/summaries/monthly

NOAA Climate Data Portal

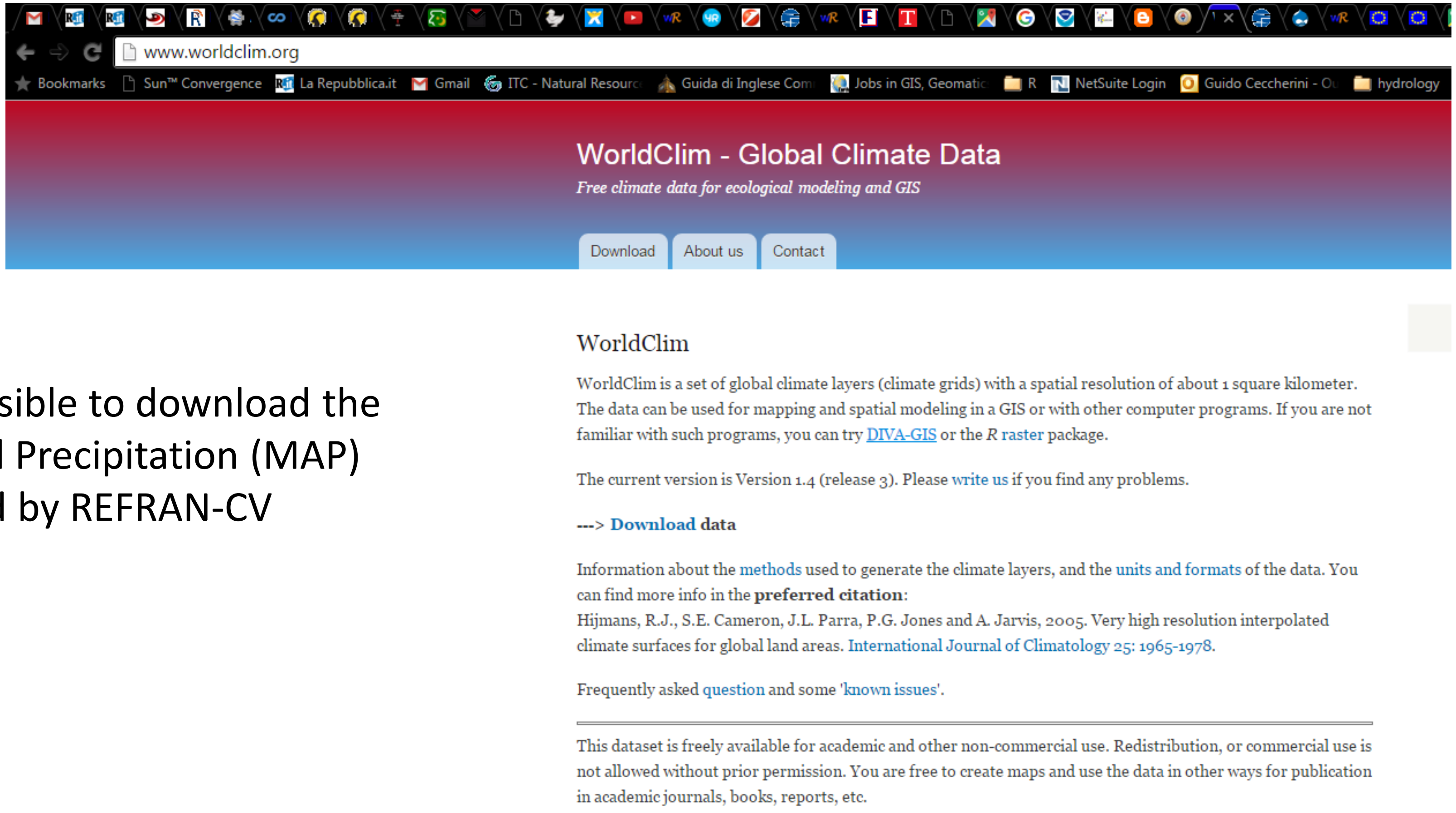
Here it is possible to download precipitation records

NOAA Climate Data Portal



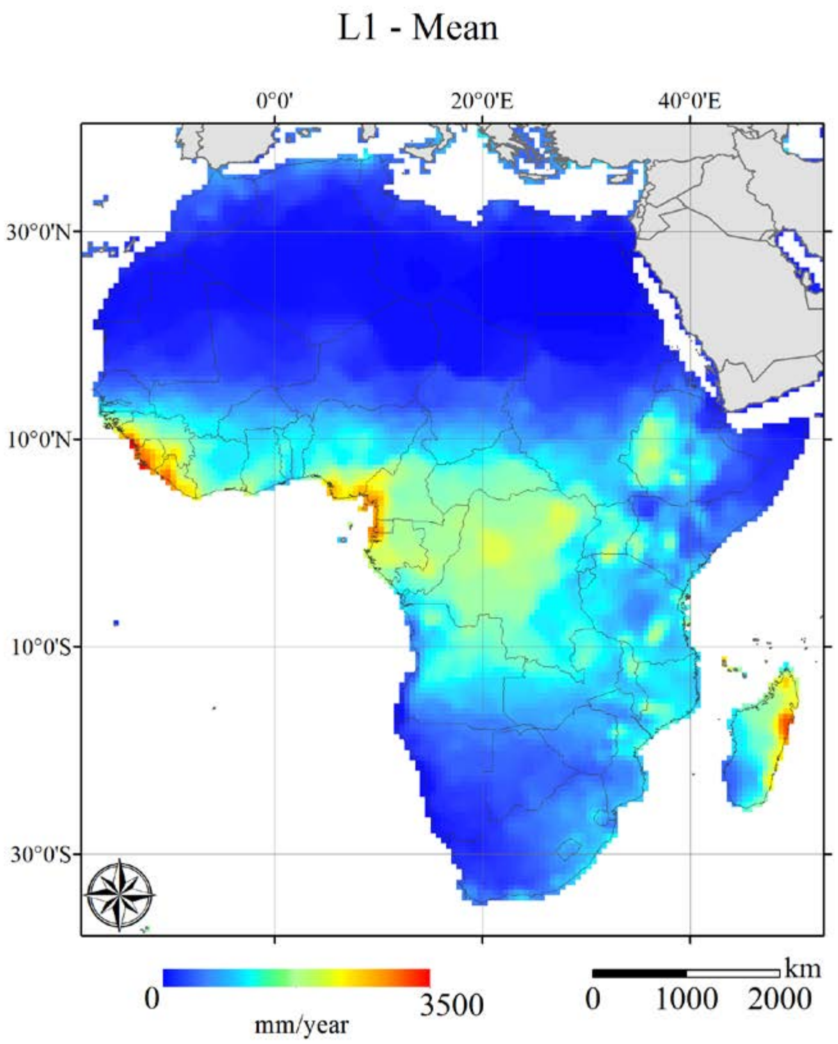
WorldClim portal

Here it is possible to download the Mean Annual Precipitation (MAP) map required by REFRAN-CV



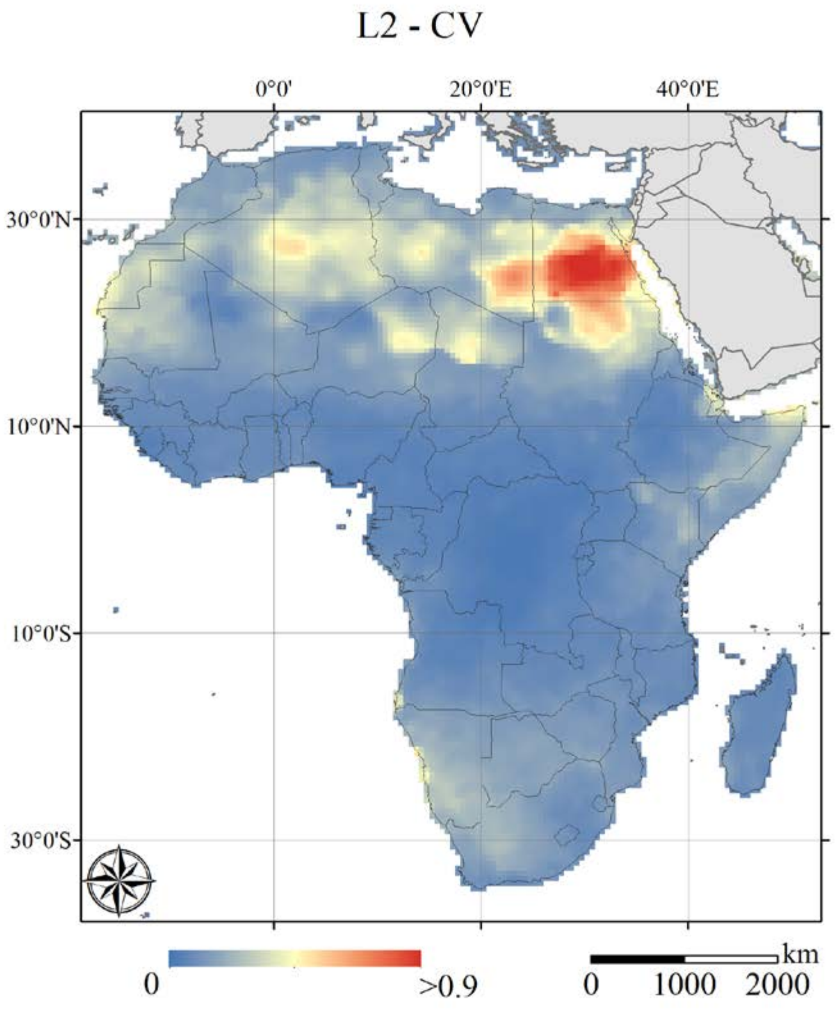
1st L-Mean

The mean total annual precipitation distribution of the first L-moment follows the well-known distribution of global total annual precipitation.



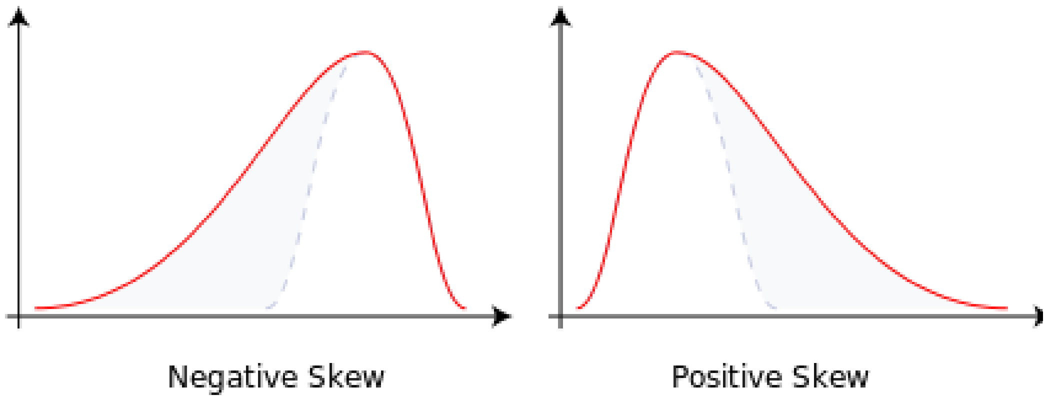
2nd Coefficient of L-variation

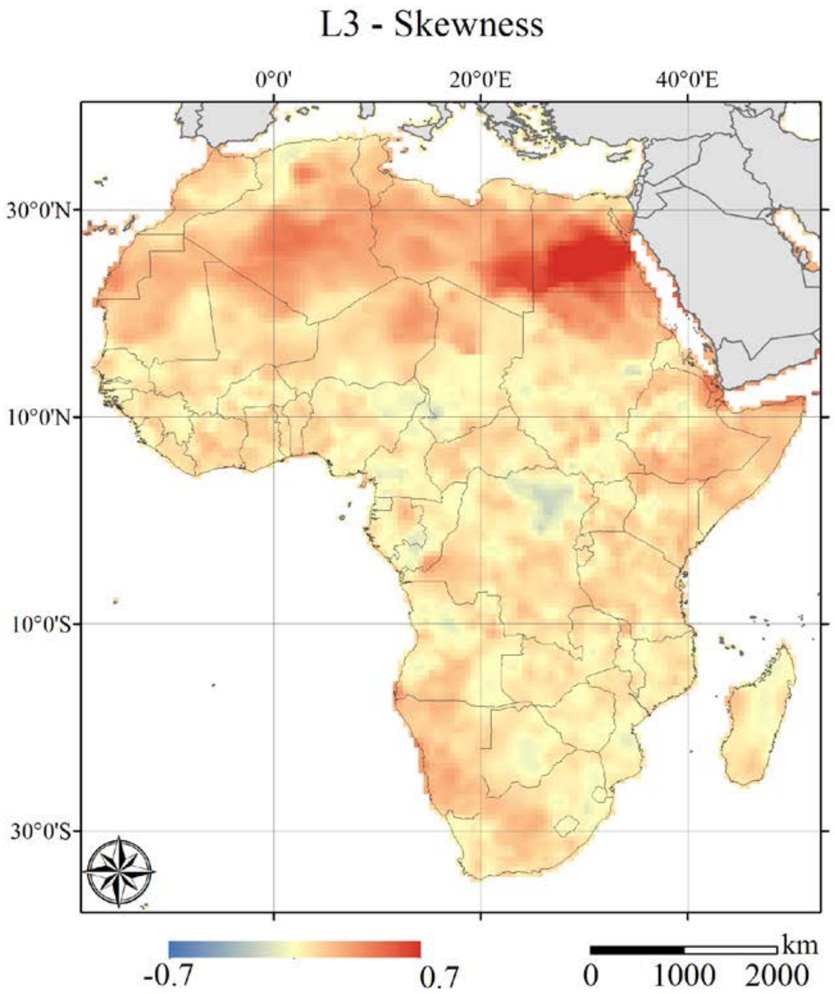
- The L-moment ratio (L-cv) measures a variable’s dispersion, i.e. the expected difference between two random samples.



3rd L-Skewness

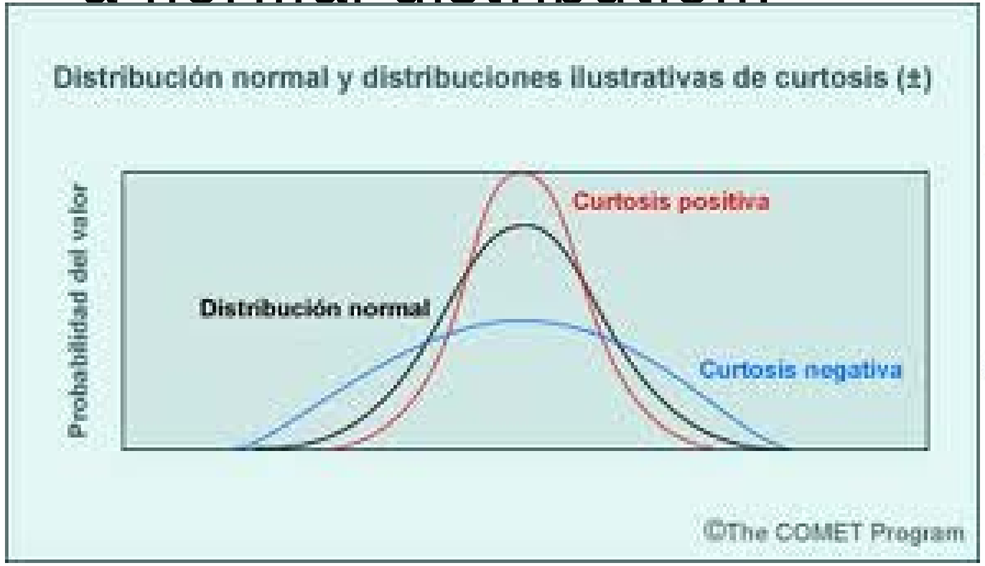
- The L-skewness quantifies the asymmetry of the samples distribution

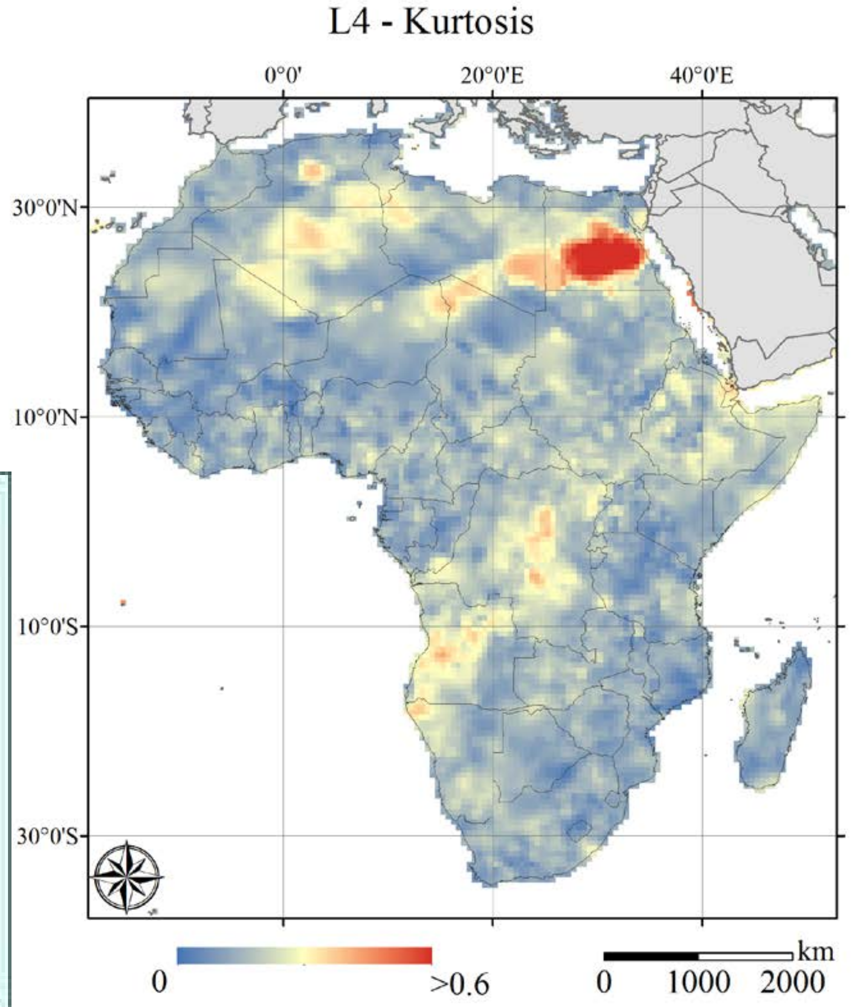




4th L-Kurtosis

- L-kurtosis measures whether the samples are peaked or flat relative to a normal distribution.



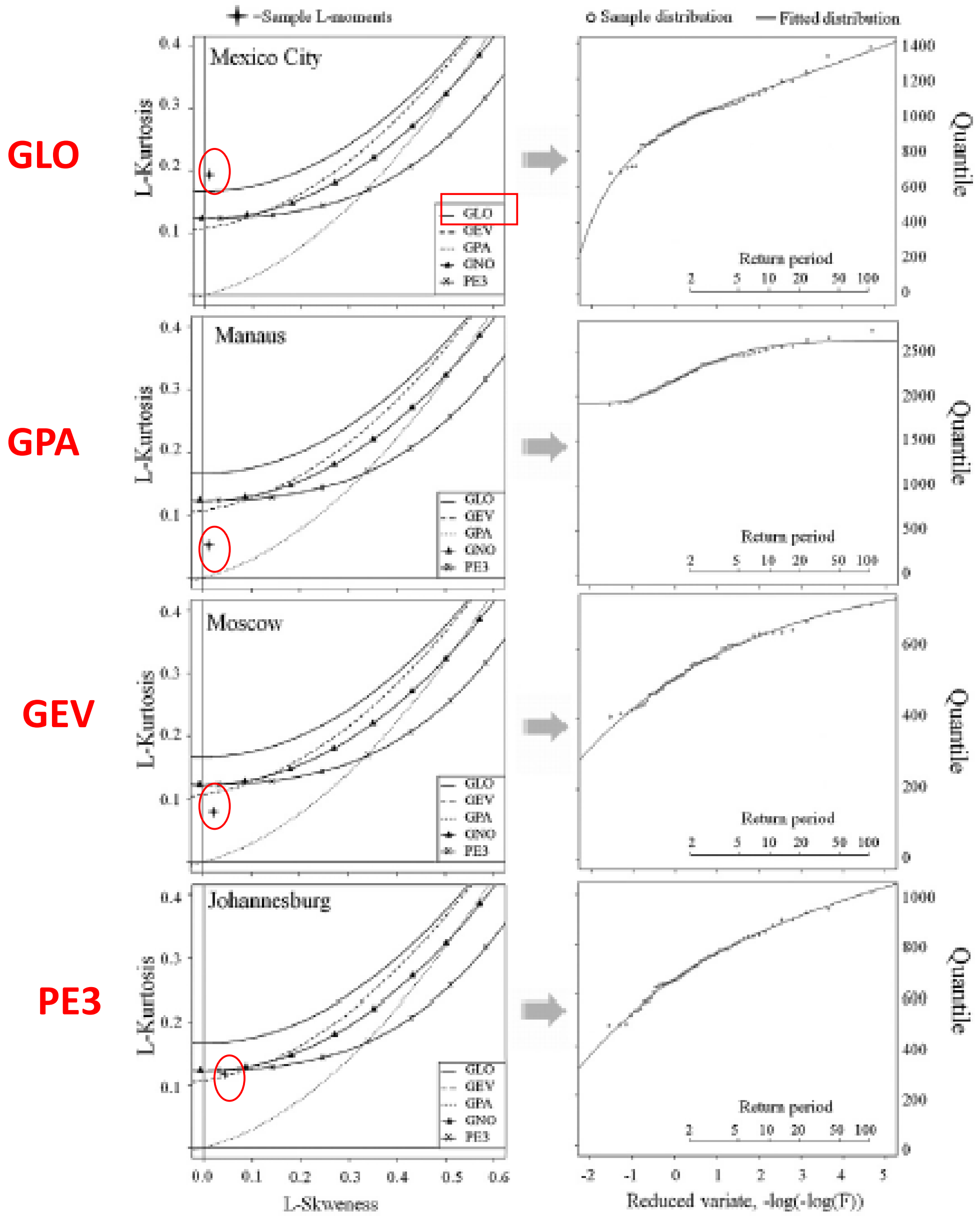


Probability distribution functions

Table 1 Name, acronym and equation of the distribution functions tested in the assessment

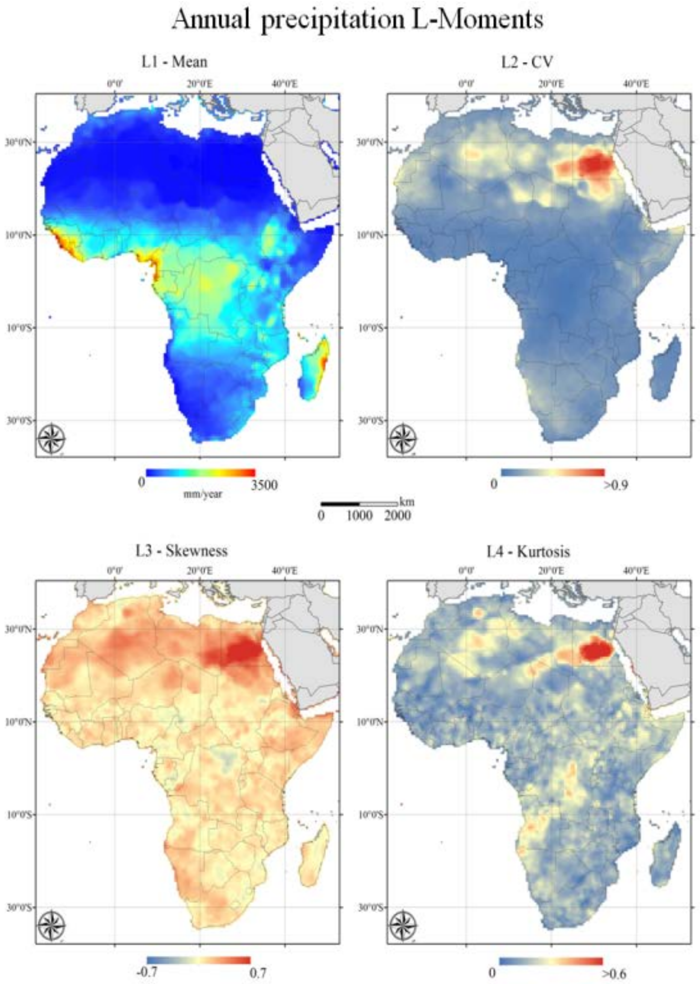
Distribution function	Acronym	Equation
Generalised logistic	GLO	$F(x) = \frac{1}{(1+\exp(-y))}$, where: $y = \left(\frac{-1}{k}\right)\log\left(\frac{1-k(x-\xi)}{\alpha}\right)$
Generalised extreme-value	GEV	$F(x) = \exp(-\exp(-y))$, where: $y = \left(\frac{-1}{k}\right)\log\left(\frac{1-k(x-\xi)}{\alpha}\right)$
Generalised normal	GNO	$F(x) = Phi(y)$, where: $y = \left(\frac{-1}{k}\right)\log\left(\frac{1-k(x-\xi)}{\alpha}\right)$
Three-parameter lognormal	PE3	$f(x) = \text{frac}\left(e^{\frac{ky-y^2}{2}}\right)(\alpha(2\pi^{-2}))$ $y = \left(\frac{-1}{k}\right)\log\left(\frac{1-k(x-\xi)}{\alpha}\right)$ if $k \neq 0$, $y = \frac{(x-\xi)}{\alpha}$ if $k = 0$
Generalised Pareto	GPA	$F(x) = 1-\exp(-y)$, where: $y = \left(\frac{-1}{k}\right)\log\left(\frac{1-k(x-\xi)}{\alpha}\right)$

Note: In the equations, ξ represents the location parameter, α is the scale parameter and k is the shape parameter of the distribution functions. *Phi*(*y*) is the distribution function of normal distribution, with x bounded by $\xi + \alpha/k$ from below if $k < 0$ and from above if $k > 0$

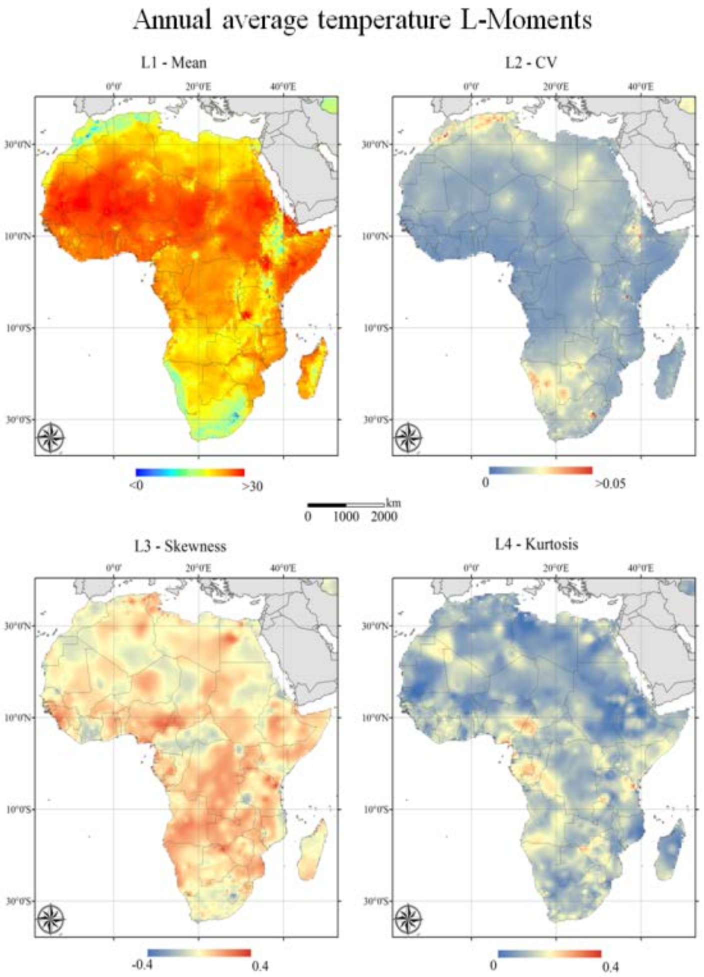


Results

Frequency analysis at annual time-scale



Available at monthly or annual scale

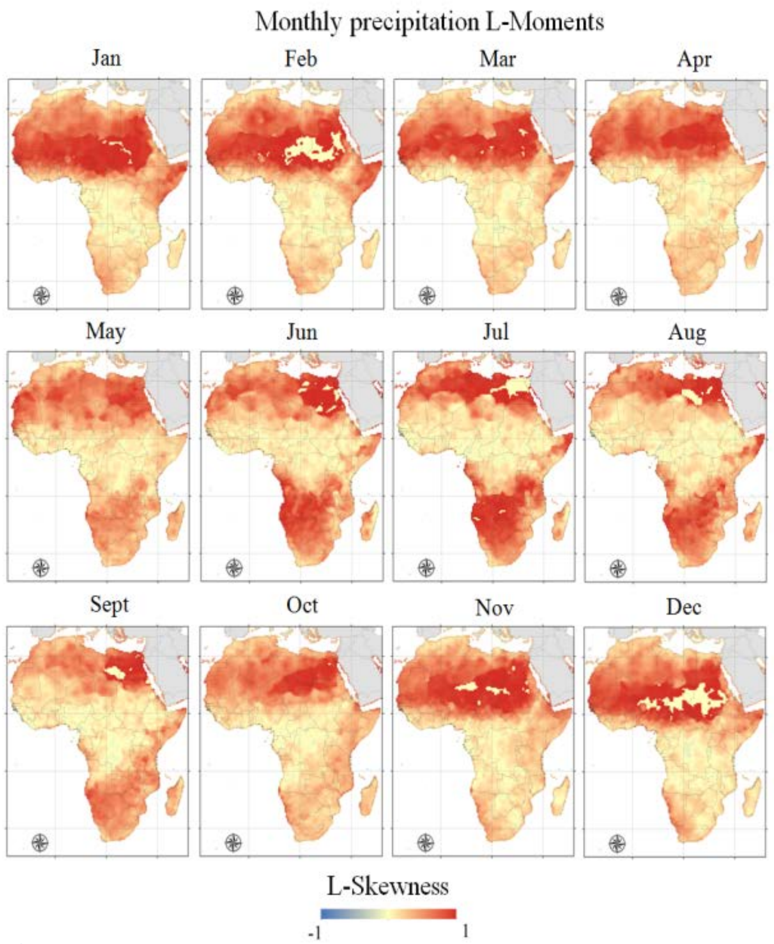


Source:
Maeda, E.E. et al "Characterization of global precipitation frequency through the L-moments approach". AREA- Royal Geographical Society

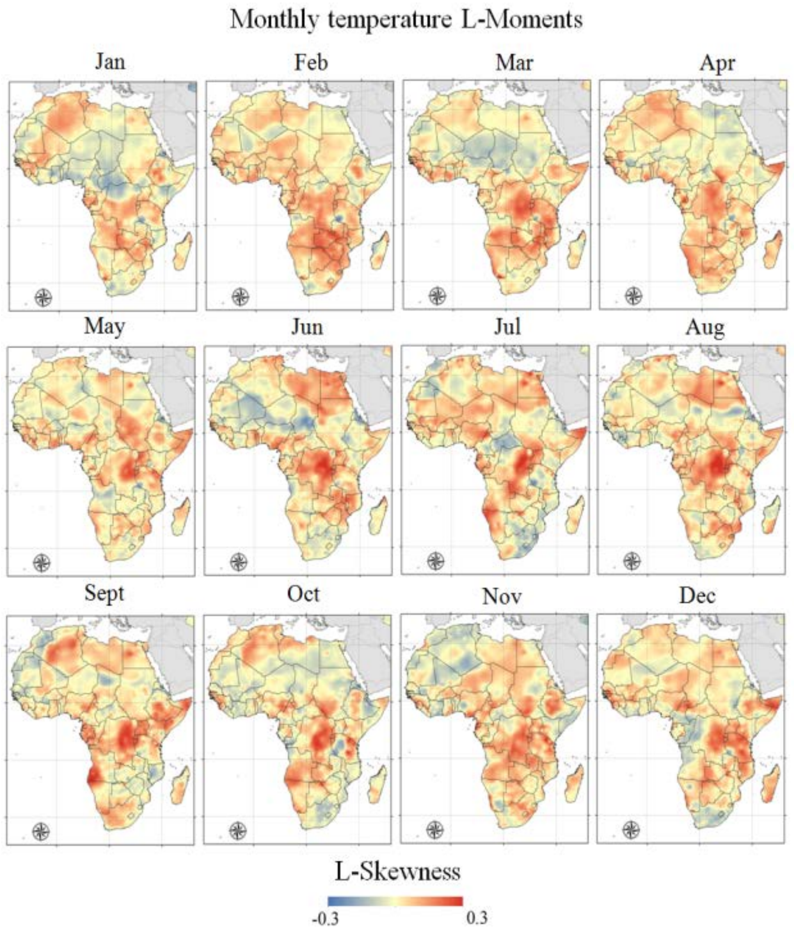
Results

Frequency analysis at monthly time-scale

Precipitation



Temperature



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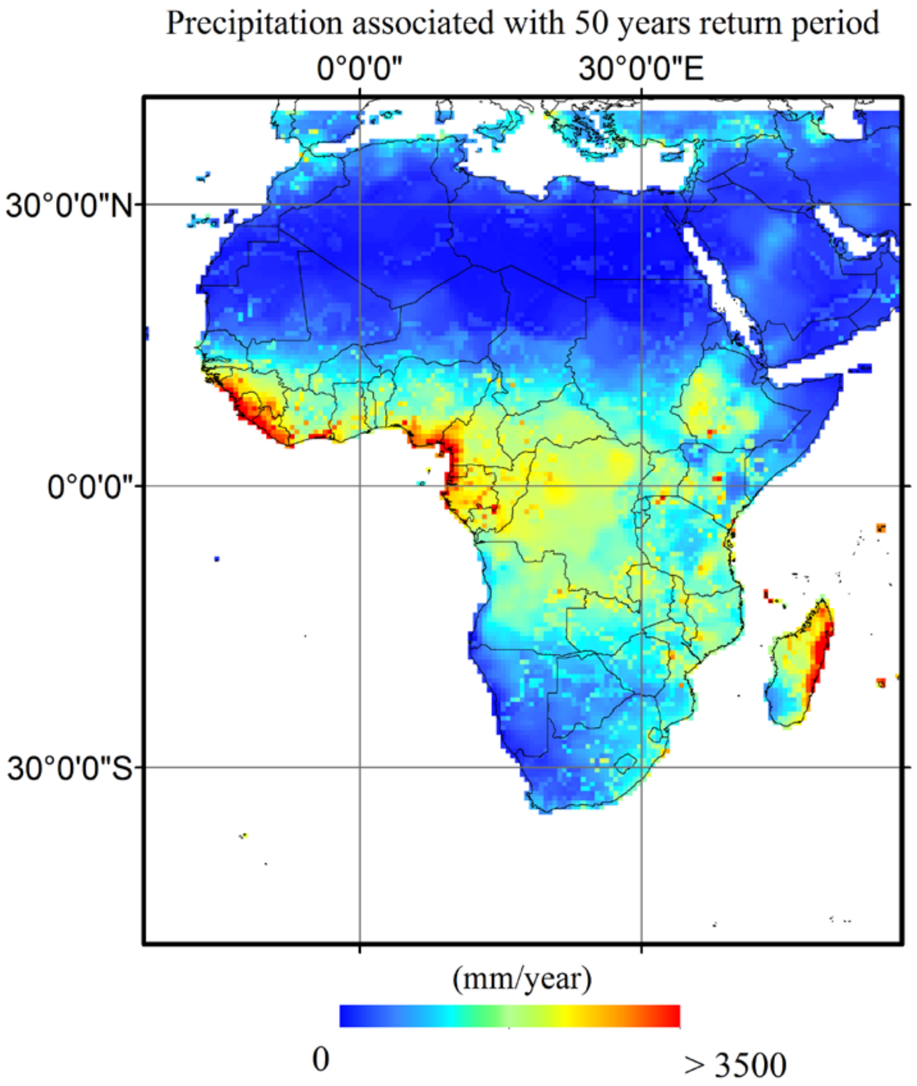
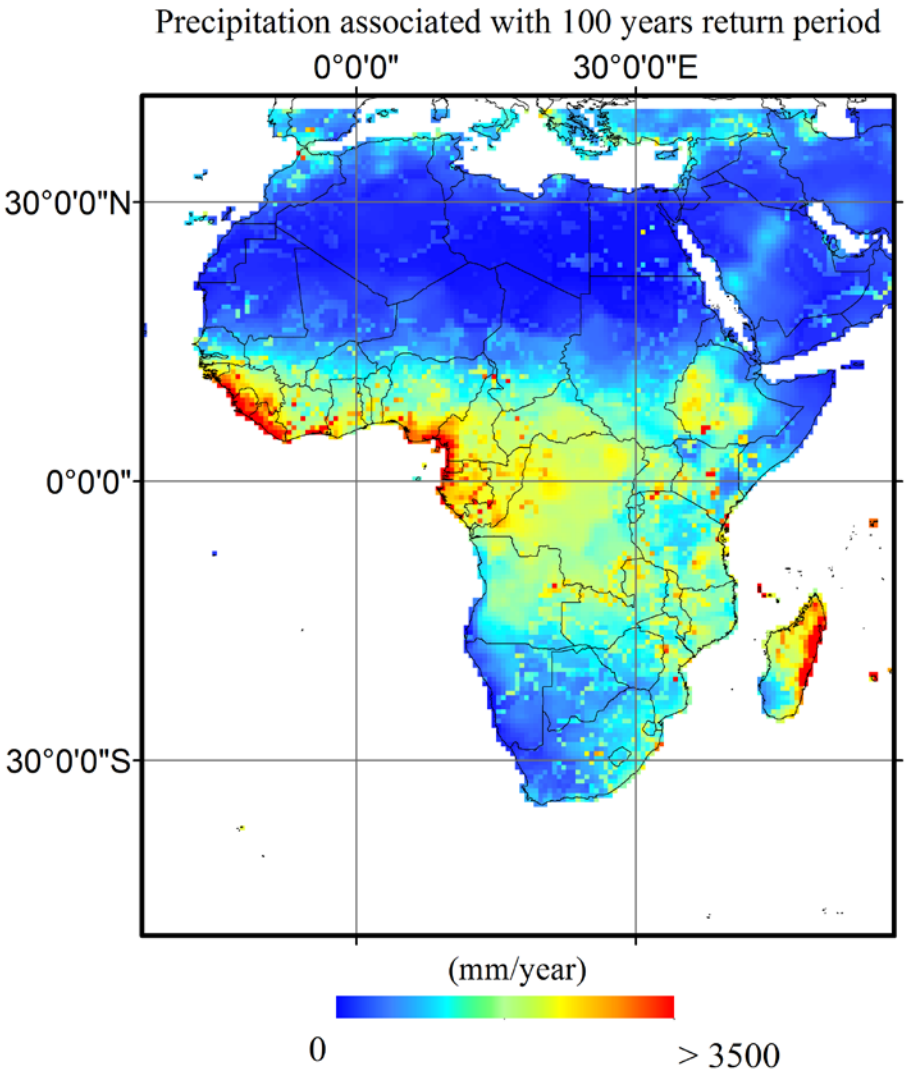
Annual precipitation return period





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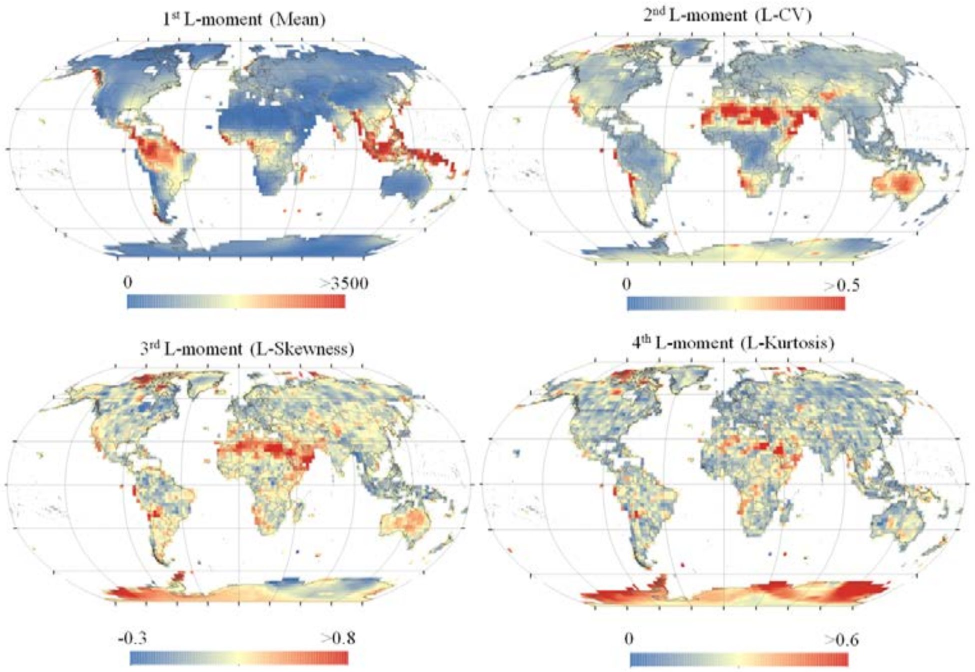

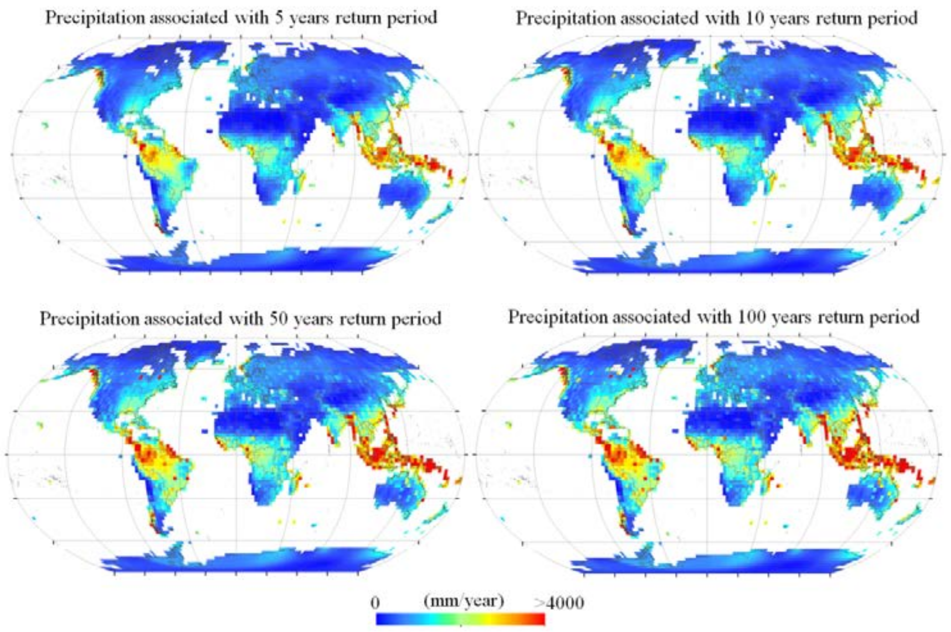

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